## How Do You Measure Up? Measuring Tree Height

## Activity Overview

Students will estimate tree height in a variety of ways.

## Objectives

Students will:

- Determine a measurement indirectly using estimation, ratio, proportion, and mean
- Measure length, height, and angle directly
- Use right angle trigonometry to solve a real world problem


## Subjects Covered

Math

## Grades

K through 12

## Activity Time

15 minutes introduction, 45 minutes in field and 15 minutes calculation/discussion in classroom

## Season

Any

## Materials

Methods 1, 2, and 3: meter stick or tape measure, calculator
Method 4: Sighting Stick (See resources below), meter stick or tape measures, calculator
Method 5: Large protractor, tape measure, meter stick, calculator, trigonometry tables (or calculator with trigonometric functions)

## State Standards

Math:
Use reasoning abilities (A.4.1, A.8.1, A.12.1)

Connect mathematical learning with other subjects (A.4.3)
Use vocabulary, symbols, notation (A.4.4)

Explain solutions to problems (A.4.5)
Communicate logical arguments (A.8.2)
Analyze non-routine problems (A.8.3)

## Background

Measuring the height of trees, buildings and other tall objects is an important part of site analysis. You should know the location, size, and height of any object on your proposed restoration site. The object may affect the plants that are growing and can grow there, the animals that will inhabit and live in the area, the temperature, the sunlight, and the soil conditions. Knowledge of how the object casts a shadow through the day and through the seasons should be considered in your planting design.

There are many ways to estimate height. Below are five methods, starting with the simplest.

## Activity Description

Estimate tree height by one of the methods described below and record your results on the student data sheet. After you finish your estimations, record your height estimate on a class data sheet. Compare other estimates and other methods. Which methods do you think are the most accurate?

## 1. Rough Estimate

Work in groups of two. Measure the height of one person and record. That person then stands straight against the tree. The second person stands at a distance and estimates how many "heights" of that person make up the tree height. Walk farther back and repeat. Make this estimate four times and find the average. Multiply that estimate by the person's measured height to get the tree height.

## Example:

Carla is $4^{\prime} 4$ " tall. Her partner estimates that the tree is 4.5 "Carla heights" tall.
$4^{\prime} 4 "=52^{\prime \prime}, 52^{\prime \prime} \times 4.5=234^{\prime \prime}$, the tree height estimate is $234^{\prime \prime}=19^{\prime} 6^{\prime \prime}$

## 2. Logger's Estimate

Work in groups of two. One person stands at a distance from the tree and extends an arm to full arm's length. Bracket the tree height between the thumb and forefinger. If the tree is too big, walk farther away from the tree. Without changing the distance between the fingers, rotate the hand so the thumb is on the base of the tree and the forefinger marks a spot along the ground some distance out from the tree base. The second person then locates the spot on the ground identified by the first person's forefinger. The distance from that spot to the base of the tree equals the height of the tree. (Note: It is important that the first person keep their arm fully extended throughout this exercise; bending your arm to make the tree "fit" between your thumb and forefinger. If the tree does not fit, walk further away or closer to the tree.)

## How Do You Measure Up? Measuring Tree Height (cont.)

Develop effective oral \& written presentations (A.8.4)

Use reason and logic (A.12.1)
Communicate logical arguments (A.12.2)
Analyze non-routine problems \& arrive at solutions (A.12.3)
Develop effective oral \& written presentations (A.12.4)
Organize work \& present mathematical procedures \& results (A.12.5)
Represent \& explain whole numbers, decimals, \& fractions (B.4.1)
Identify \& represent equivalent fractions (B.4.4)

Select \& use appropriate computational procedures (B.4.5)
Perform \& explain operations on rational numbers (B.8.2)
Use appropriate computational procedures with rational numbers (B.8.7)
Create \& critically evaluate numerical arguments (B.12.5)
Routinely assess the acceptable limits of error (B.12.6)
Describe simple two-\& three-dimensional figures (C.4.1)
Identify \& use relationships among figures (C.4.3)

Identify \& demonstrate an understanding of trigonometry (C.12.5)
Recognize \& describe measurable attributes \& units (D.4.1)
Demonstrate understanding of measurement (D.4.2)
Read \& interpret measuring instruments (D.4.3)

Determine measurements by using standard tools (D.4.4)
Determine measurements by using basic relationships or estimations (D.4.5)
Identify \& describe attributes in situations not directly or easily measurable (D.8.1)
Demonstrate understanding of measurement facts, principles, techniques (D.8.2)
Determine measurement directly by using standard units (D.8.3)
Determine measurement indirectly (D.8.4)

## 3. Shadows

Work in groups of two. Measure the height of one person and then measure their shadow and record. Immediately measure the length of the tree's shadow and record. The tree height can be calculated by the following proportion:

$$
\frac{\begin{array}{l}
\text { height of tree } \\
\text { length of tree's shadow }
\end{array}=\quad \underline{\text { height of person }}}{\text { length of person's shadow }}
$$

## Example:

Willy is $5^{\prime} 1^{\prime \prime}\left(61^{\prime \prime}\right)$ tall and his shadow measures $3^{\prime} 10^{\prime \prime}\left(46^{\prime \prime}\right)$. The tree shadow measures $21^{\prime} 7^{\prime \prime}\left(259^{\prime \prime}\right)$. Therefore the following ratio is set up:
$\frac{\mathrm{X}^{\prime \prime}}{259^{\prime \prime}} \quad=\quad \frac{61^{\prime \prime}}{46^{\prime \prime}}$

The tree height (X) equals $343^{\prime \prime}$ or $28^{\prime} 7^{\prime \prime}$

## 4. Sighting Stick

Work in groups of two. Using the sighting stick, move a distance from the tree so that, when held at arm's length, the base of the stick is at the base of the tree and the stick just covers the height of the tree. In this position, sight through the hole to the tree. The second person should mark the spot on the tree that is visible through the hole. Because the stick is 30 cm long and the hole is drilled at 3 cm inch, tree height can be calculated using the following proportion:

$$
\frac{3 \mathrm{~cm}}{30 \mathrm{~cm}}=\frac{\text { height of the marked spot on the tree }}{\text { height of tree }}
$$

## 5. Trigonometry <br> From the Ground:

Work in pairs. Pick a spot at some distance from the tree. Measure and record the distance from the base of the tree to that spot. Measure the angle necessary to sight to the top of the tree from that spot on the ground. To have a more accurate measurement use a large protractor on the ground, site to the top of the tree using a meter stick and read off the angle (see Figure 1).

## From Standing Position:

Alternatively, you may sight from a standing position. In order to do this with more accuracy, devise a crude clinonmeter. You can do this by attaching a straw length-wise on the flat side of a protractor and then tape a string with a weight on the end to the middle of the straw. The weight should swing freely. By sighting to the tree top along the base of the protractor (through the straw), the string will then read off the correct angle. After cal-

## How Do You Measure Up? Measuring Tree Height (cont.)

Determine measurement indirectly (D.12.3)

Describe a set of data (E.4.2)
Analyze information from organized \& displayed data (E.8.3)
Use results of data analysis (E.8.4)
culating the tree height, add to that figure, your height up to your eye.

Tree height can be calculated using the following equation:
tangent of an angle $=$ length of opposite side/ height of tree $=$ tree height
length of adjacent side/ distance to tree
distance to tree

## Example:

Let's say the angle measured to the top of the tree is sixty degrees and you are standing twenty five feet from the tree when you measured. The variable X will represent the unknown height of the tree.

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tangent 60 X 25 feet = 43.3 feet
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The tree height $(\mathrm{X})$ will equal 43 feet and three inches tall.
You will probably need a scientific calculator to figure this out. If you don't have one readily available, the Calculator on Windows has an option under "View" for "Scientific". Click that setting (not Standard). Also, make sure to click "Degrees" (not Radians or Grads).

For this example you will:
Punch in 60, press the "tan" button, then hit the multiplication button and finally punch 25 . Hit the equal sign to get your answer. If you have done it correctly it should say 43.3 (feet).

## Extensions

- Measure shadows at different times of year and record or map measurements. Trace the change in shadows and relate to the sun's position in the sky.
- See Earth Partnership for Schools activity, "Measuring Up Tree Size" to learn how to measure trunk diameter and crown size, then calculate tree size.
- Keep a record of trees growing on the school site.
- Invite a forester into the classroom to learn about modern tree measurement techniques and how this information is used in forestry.


## Additional Resources

## Websites

- National arbor day foundation. http://www.arborday.org/kids/TeachingYouth.cfm
- The wonderful world of trees. http://www.domtar.com/arbre/english/start.htm
- American forests. http://www.americanforests.org/resources/bigtrees/
- Project learning tree. http://www.plt.org/
- Science Friday: Making science radioactive. http://www.sciencefriday.com/kids/sfkc20040423-1.html\#activities


## Supplies

- Sighting Stick - To make a sighting stick, cut a $3-4 \mathrm{~cm}$ wide strip of wood to a height of exactly 30 cm . Drill a hole at exactly 3 cm . The hole should be $5-10 \mathrm{~mm}$ diameter.


## How Do You Measure Up? Measuring Tree Height (cont.)

## Assessments

- Based on the data collected, which tree height measurements seem the most accurate and why?
- Based on the data collected, which tree height measurement is the least accurate and why?
- Determine the range, medium, and mode for the tree height data collected.

Figure 1


Figure 2


Height of Person

## Tree Height Data Sheet

## Rough Estimate

tree height $=$ height of person X number of "heights" (use average)
$\begin{array}{llll}\text { Trial } 1 & \text { Trial } 2 & \text { Trial } 3 & \text { Trial } 4\end{array}$
number of "heights" of person make up the tree
average of trials $\qquad$ X height of person $\qquad$ =
height of tree $\qquad$

## Logger's Estimate

height of tree $\qquad$

## Shadows

height of tree $=$ height of person
length of tree's shadow length of person's shadow
Therefore,
height of tree $=$ height of person $X$ length of tree's shadow
length of person's shadow
height of person
length of tree's shadow $\qquad$
length of person's shadow $\qquad$
height of tree $\qquad$

## Sighting Stick

$3 \mathrm{~cm}=$ height of the marked spot on the tree 30 cm height of tree

Therefore,

$$
\text { height of tree }=\frac{30 \mathrm{~cm} \mathrm{X} \mathrm{height} \mathrm{of} \mathrm{the} \mathrm{marked} \mathrm{spot} \mathrm{on} \mathrm{the} \mathrm{tree}}{3 \mathrm{~cm}}
$$

height of tree $\qquad$

## Trigonometry

tan of angle $=$ length of leg opposite angle $=$ tree height length of leg adjacent angle distance to tree
Therefore,
tree height $=\tan$ of angle X distance to tree
distance to tree $\qquad$
angle $\qquad$
$\tan$ of angle $\qquad$ height of tree $\qquad$

